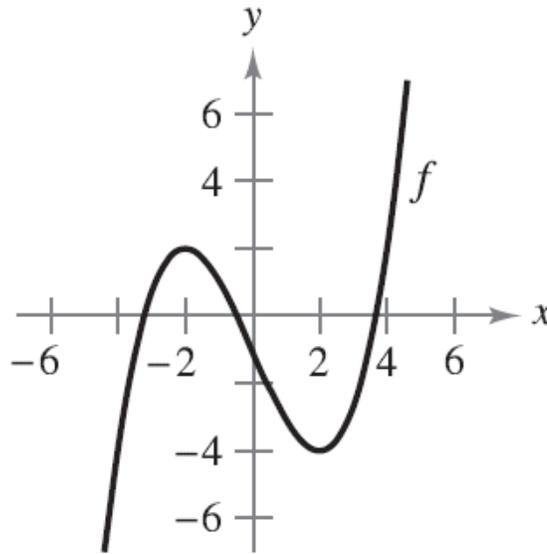


AP Calculus AB / BC  
Distant Online Assignment 6-1: Applications of Derivatives

1. Show that if  $x = c$  is a critical number of the function  $f$ , then it is also a critical number of the function  $g(x) = f(x) + k$ , where  $k$  is a constant.
2. Let  $p(x) = Ax^2 + Bx + C$ , where  $A \neq 0$  and  $A, B, C$  are constants. Show for any interval  $[a, b]$ , the value of  $c$  guaranteed by the Mean Value Theorem is  $\frac{b-a}{2}$ .
3. Use the definition of increasing and decreasing to show that the function  $f(x) = \frac{1}{x}$  is decreasing on the interval  $x > 0$ .
4. Let  $f$  and  $g$  be differentiable functions where  $f''(x) \neq 0$  and  $g''(x) \neq 0$ . Show that if  $f$  and  $g$  are concave upward on the interval  $(a, b)$ , then  $f + g$  is also concave upward on  $(a, b)$ .

For questions 5-7, the graph of  $f$  is given below.



5. Determine the x-values where:
  - a.  $f'(x) = 0$ . Justify your answer.
  - b.  $f''(x) = 0$ . Justify your answer.
6. Determine the intervals where:
  - a.  $f'(x) > 0$ . Justify your answer.
  - b.  $f'(x) < 0$ . Justify your answer.
7. Determine the intervals where:
  - a.  $f''(x) > 0$ . Justify your answer.
  - b.  $f''(x) < 0$ . Justify your answer.

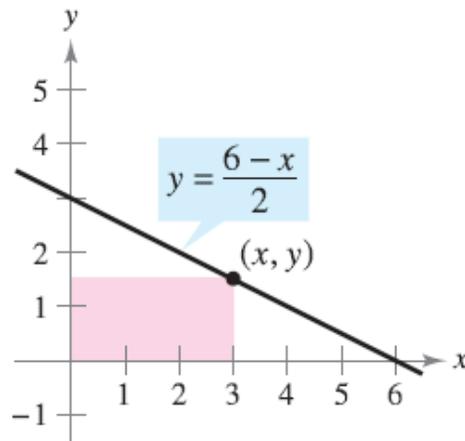
AP Calculus AB / BC  
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8. Let  $y = f(x)$  be the particular solution to the differential equation  $\frac{dy}{dx} = \frac{2x+12}{3y^2-4}$  with the initial condition  $f(1) = 2$ .

- Determine if  $f$  is increasing or decreasing at  $x = 1$ . Justify your answer.
- Determine if  $f$  is concave up or concave down at  $x = 1$ . Justify your answer.
- Write the equation of the tangent line to the graph of  $f$  at  $x = 1$ .
- Use the equation of the tangent line to approximate  $f(1.2)$ . Determine if the approximation is less than or greater than  $f(1.2)$ . Justify your answer.

9. A balloon rises at a rate of 4 meters per second from a point on the ground 50 meters from an observer. Find the rate at which the angle of elevation of the observer is changing when the balloon is 50 meters above the ground.

10. A rectangle is bounded by the  $x$ -axis and  $y$ -axis and the graph of  $y = \frac{6-x}{2}$  in the figure below.



Find the length and width of the rectangle that will yield maximum area.